Multicriteria Risk-averse Optimization in Humanitarian Relief Network Design

Simge Küçükyavuz

Department of Industrial Engineering and Management Sciences

Northwestern University

Joint work with **Nilay Noyan and Merve Meraklı**

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In Memoriam

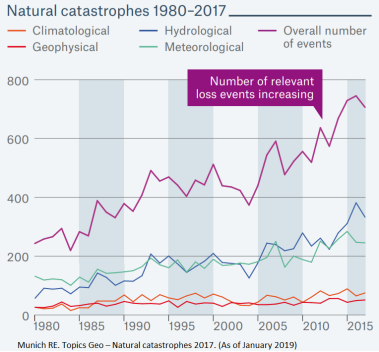
Agenda

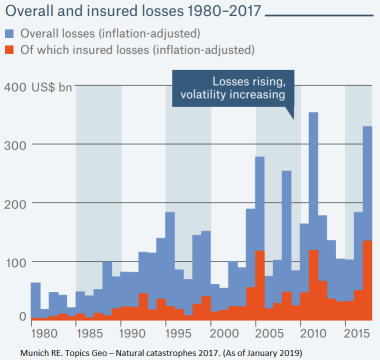
- Motivation: Risk-averse decision making in humanitarian logistics
- Two-stage stochastic programming under multivariate risk constraints
  - Delayed Cut Generation for Deterministic Equivalent Formulation (DCG-DEF)
  - Delayed Cut Generation with Scenario Decomposition (DCG-SD)
- Application to a pre-disaster relief network design problem
- Concluding remarks

Natural disasters: Putting things into perspective...

1999 İzmit Earthquake in Turkey: More than 17,000 fatalities and an estimated 500,000 left homeless.

"In 2017, 335 natural disasters affected over 95.6 million people, killing an additional 9,697 and costing a total of US $335 billion." CRED, Annual Disaster Statistical Review 2017.

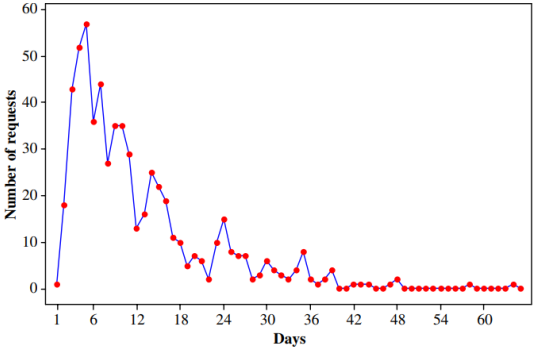




Need for efficient and effective disaster relief systems!

Natural disasters: Putting things into perspective...

- Large volumes of demand for relief items (medical supplies, water, food etc.) in the immediate aftermath of disaster.
- Transportation network could be severely damaged.



Number of requests for critical supplies after Hurricane Katrina (Holguín-Veras and Jaller, 2012)

Pre-disaster relief network design problem

- Common strategy: pre-positioning of relief supplies at strategic locations to improve the effectiveness of the immediate post-disaster response operations.
- At the time of decision making, there is **high level of uncertainty** in supply (undamaged pre-stocked supplies), demand, and transportation network conditions.
- **Multiple critical issues** for pre-disaster relief network design (aside from cost):
  - Meeting basic needs of most of the affected population,
  - Ensuring accessibility of the relief supplies,
  - Ensuring equity in supply allocation, etc.

- **Multiple stakeholders** with different perspectives
  - government, community, non-governmental organizations (NGOs), engineers, sponsors etc.

Risk-averse stochastic optimization in humanitarian logistics

- Would the decision makers be indifferent to a small probability that
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  - Certain locations do not have access to medical care?

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- Risk-averse stochastic optimization problems provide the flexibility to capture a wider range of risk attitudes.

Risk-averse stochastic optimization in humanitarian logistics

- Would the decision makers be indifferent to a small probability that
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  - Certain locations do not have access to medical care?
- Need to consider a wide range of possible outcomes, not just the most likely or worst-case scenario, for multiple sources of risk.
- Risk-averse stochastic optimization problems provide the flexibility to capture a wider range of risk attitudes.
- We aim to propose **risk-averse** optimization models and solution algorithms that consider
  - **Multiple sources of risk** (cost, accessibility, equity)
  - **A group of decision makers with different opinions on the importance (weight) of each criteria** (government, community, NGOs)

  in a **two-stage decision making** framework.

Two-Stage Decision Making Framework

- First-stage actions (x) $\rightarrow$ observe randomness $\rightarrow$ Second-stage actions (y)
  (Pre-disaster) (Disaster) (Post-disaster)

Two-Stage Decision Making Framework

- First-stage actions (x) $\rightarrow$ observe randomness $\rightarrow$ Second-stage actions (y)
  (Pre-disaster) (Disaster) (Post-disaster)
- The first-stage decisions are made before the uncertainty is resolved.
  - e.g., humanitarian relief facility location and inventory level decisions.
- The second-stage (recourse) decisions are made after the uncertainty is resolved. Represent the operational decisions, which depend on the realized values of the random data.
  - e.g., distribution of the relief supplies (allocation decisions).

Two-Stage Decision Making Framework

- First-stage actions (x) $\rightarrow$ observe randomness $\rightarrow$ Second-stage actions (y)
  (Pre-disaster) (Disaster) (Post-disaster)
- A finite probability space $(\Omega, 2^{\Omega}, \Pi)$ with $\Omega = \{\omega_1, \ldots, \omega_m\}$ and $\Pi(\omega_s) = p_s, s \in S := \{1, \ldots, m\}$.

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- A finite probability space $(\Omega, 2^{\Omega}, \Pi)$ with $\Omega = \{\omega_1, \ldots, \omega_m\}$ and $\Pi(\omega_s) = p_s, s \in S := \{1, \ldots, m\}$.
- The general form of a risk-neutral two-stage stochastic programming model:

$$\min_{\mathbf{x} \in \mathcal{X}} f(\mathbf{x}) + \mathbb{E}(Q(\mathbf{x}, \boldsymbol{\xi}(\omega)))$$

$$Q(\mathbf{x}, \boldsymbol{\xi}(\omega_s)) = \min_{\mathbf{y}_s \in \mathcal{Y}(\mathbf{x}, \boldsymbol{\xi}(\omega_s))} \mathbf{q}_s^{\top} \mathbf{y}_s, \quad \mathcal{Y}(\mathbf{x}, \boldsymbol{\xi}(\omega_s)) = \{\mathbf{y}_s \in \mathbb{R}_+^{n_2} : T_s \mathbf{x} + W_s \mathbf{y}_s \geq \mathbf{h}_s\}$$

Two-Stage Stochastic Programming with Multivariate Stochastic Preference Constraints

Given

- a benchmark (reference) random outcome vector $\mathbf{Z} \in \mathbb{R}^d$
- multivariate risk-based preference relation $\succcurlyeq$
  ("A $\succcurlyeq$ B" implies A is preferable to B w.r.t. its risk)

A class of risk-averse two-stage optimization problems:

$$\begin{aligned} \min \quad & f(\mathbf{x}) + \mathbb{E}(Q(\mathbf{x}, \boldsymbol{\xi}(\omega))) \\ \text{s.t.} \quad & \hat{\mathbf{G}}(\mathbf{x}, \mathbf{y}) \succcurlyeq \mathbf{Z}, \\ & \mathbf{x} \in \mathcal{X}, \\ & Q(\mathbf{x}, \boldsymbol{\xi}(\omega_s)) = \mathbf{q}_s^\top \mathbf{y}(\omega_s), \quad \forall s \in S, \\ & \mathbf{y}(\omega_s) \in \mathcal{Y}(\mathbf{x}, \boldsymbol{\xi}(\omega_s)), \quad \forall s \in S. \end{aligned}$$

Decision-based d-dimensional random outcome vector:

$$\left[\hat{\mathbf{G}}(\mathbf{x}, \mathbf{y})\right](\omega) = \hat{\mathbf{G}}(\mathbf{x}, \mathbf{y}(\omega), \boldsymbol{\xi}(\omega)).$$

How to measure risk?

Conditional Value-at-Risk (CVaR) for assessing the univariate risk

- Desirable risk measure (Artzner et al., 1999; Kusuoka, 2001)
- Easy to incorporate into optimization problems (convexity)

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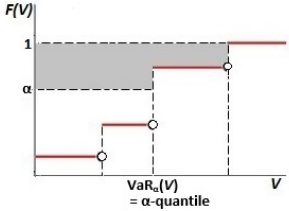
V: univariate random variable that takes value v_i with probability p_i, $i \in [n] := \{1, \ldots, n\}$. For confidence level α,



$$\mathrm{CVaR}_\alpha(V) = \inf\left\{\eta + \frac{1}{1-\alpha}\mathbb{E}([V-\eta]_+), \eta \in \mathbb{R}\right\},$$

$$= \min \quad \eta + \frac{1}{1-\alpha}\sum_{i=1}^{n} p_i w_i$$

$$\text{s.t.} \quad w_i \geq v_i - \eta, \; w_i \geq 0, \; i \in [n].$$

- $\mathrm{CVaR}_\alpha(V)$: "If we are unlucky and do worse than α-quantile, then the expected outcome is $\mathrm{CVaR}_\alpha(V)$."

Challenges

- How to define vector-valued risk?
  - Vector-valued VaR (p-efficient points) well-defined (Prekopa)
  - Meraklı and K. (2018) discuss issues with existing definitions of vector-valued CVaR (e.g., CVaR$<$VaR in some definitions)
  - and propose a new definition

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- How to define vector-valued risk?
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  - and propose a new definition – Hard to compute
- Need to compare two (decision-dependent) random vectors?

Multivariate Stochastic Preference Relations

Establish preference relations between two d-dimensional random vectors

$$(X_1, X_2, \ldots, X_d) \succcurlyeq (Z_1, Z_2, \ldots, Z_d)$$

- Considering multiple risk factors, a natural approach is to use univariate preference relations under a *scalarization scheme*.
  - Linear scalarization for random vector $\mathbf{X} \in \mathbb{R}^d$ based on vector $\mathbf{c} \in \mathbb{R}^d$:

$$\mathbf{c}^\top \mathbf{X} = c_1 X_1 + \cdots + c_d X_d \quad \succcurlyeq \quad \mathbf{c}^\top \mathbf{Z}.$$

  - c_i: relative importance of criterion i
- How to choose **one c**?

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  - c_i: relative importance of criterion i
- How to choose **one c**? Relative importance of risk factors ($\mathbf{c}$) is usually **ambiguous** and may be **inconsistent** especially in the presence of multiple decision makers.

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- Extend univariate stochastic preference relations to the multivariate case by considering **a family of scalarization vectors**, $\mathcal{C}$.

Multivariate Stochastic Preference Relations

Establish preference relations between two d-dimensional random vectors

$$(X_1, X_2, \ldots, X_d) \succcurlyeq (Z_1, Z_2, \ldots, Z_d)$$

- Extend univariate stochastic preference relations to the multivariate case by considering **a family of scalarization vectors**, $\mathcal{C}$.
- Require that the scalarized versions of vector-valued random variables conform to some scalar-based preference relation (Dentcheva and Ruszczynski, 2009).

$$\text{Linear scalarization: } \mathbf{c}^\top \mathbf{X} = c_1 X_1 + \cdots + c_d X_d \quad \succcurlyeq \quad \mathbf{c}^\top \mathbf{Z} \quad \forall\, \mathbf{c} \in \mathcal{C}$$

- Coefficient c_i represents subjective importance of criteria i, $i \in \{1, \ldots, d\}$.
- Set $\mathcal{C}$ corresponds to **different opinions of possibly multiple decision makers** on the relative importance of criteria.

Multivariate Stochastic Preference Relations

Establish preference relations between two d-dimensional random vectors

$$(X_1, X_2, \ldots, X_d) \succcurlyeq (Z_1, Z_2, \ldots, Z_d)$$

- Our main focus: Univariate risk measure - **Conditional value-at-risk (CVaR)** (Rockafellar and Uryasev, 2000, 2002).
- Polyhedral multivariate CVaR relation (Noyan and Rudolf, 2013): Given a set of scalarization vectors $\mathcal{C} \subset \mathbb{R}^d$ and a confidence level $\alpha \in [0, 1)$, $\mathbf{X}$ is preferable to $\mathbf{Z}$ if

$$\mathrm{CVaR}_\alpha(\mathbf{c}^\top \mathbf{X}) \leq \mathrm{CVaR}_\alpha(\mathbf{c}^\top \mathbf{Z}), \quad \forall\, \mathbf{c} \in \mathcal{C}.$$

- We introduce a new class of risk-averse two-stage stochastic programming problems with polyhedral multivariate CVaR constraints.

Related Studies on Optimization with the Multivariate Stochastic Benchmarking Constraints

- **Multivariate second-order stochastic dominance(SSD)-constrained problems** gained more attention in the literature (see, e.g., Dentcheva and Ruszczynski, 2009; Homem-de-Mello and Mehrotra, 2009; Hu et al., 2012; Dentcheva and Wolfhagen, 2015, 2016).
- SSD-based models are conservative and demanding, often leading to infeasibilities.
- A few studies consider **CVaR-based models** as a natural relaxation of the SSD-based models (see, e.g., Noyan and Rudolf, 2013; Liu et al., 2016).
- Some of the recent studies consider both (Küçükyavuz and Noyan, 2016; Noyan and Rudolf, 2016).
- All except Dentcheva and Wolfhagen (2015) study **single-stage (static)** decision making problems.

Problem Formulation

$\mathbf{x}$, $\mathbf{y}$: first- and second-stage decision vectors, respectively

$\boldsymbol{\xi}(\omega)$: random vector defined on a finite probability space with $\Omega = \{\omega_i : i \in S\}$

$\hat{\mathbf{G}}(\mathbf{x}, \mathbf{y})$: the d-dimensional random outcome vector representing the multiple random performance measures of interest associated with the first- and second-stage decisions

$\mathbf{Z}$: a given benchmark (reference) random outcome vector

$\mathcal{C}$: a given scalarization set, i.e. $\mathcal{C} \subseteq \mathcal{C}_f = \{\mathbf{c} \in \mathbb{R}^d_+ \mid \sum_{i=1}^d c_i = 1\}$

$$
\begin{aligned}
\min \quad & f(\mathbf{x}) + \sum_{s \in S} p_s Q(\mathbf{x}, \boldsymbol{\xi}(\omega_s)) \\
\text{s.t.} \quad & \mathrm{CVaR}_\alpha(\mathbf{c}^\top \hat{\mathbf{G}}(\mathbf{x}, \mathbf{y})) \leq \mathrm{CVaR}_\alpha(\mathbf{c}^\top \mathbf{Z}), \quad \forall\, \mathbf{c} \in \mathcal{C}, \\
& \mathbf{x} \in \mathcal{X}, \\
& Q(\mathbf{x}, \boldsymbol{\xi}(\omega_s)) = \mathbf{q}_s^\top \mathbf{y}(\omega_s), \quad \forall s \in S, \\
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Challenge: Infinitely many CVaR constraints.

Towards a Tractable Formulation

- It is sufficient to consider CVaR benchmarking constraints for a finite subset of $\mathcal{C}$ (Noyan and Rudolf, 2013).

$$\mathrm{CVaR}_\alpha(\mathbf{c}_{(l)}^\top \hat{\mathbf{G}}(\mathbf{x}, \mathbf{y})) \leq \mathrm{CVaR}_\alpha(\mathbf{c}_{(l)}^\top \mathbf{Z}), \quad l = 1, \ldots, \bar{L},$$

  where $\mathbf{c}_{(1)}, \mathbf{c}_{(2)}, \ldots, \mathbf{c}_{(\bar{L})}$ are projected extreme points of a higher dimensional polyhedron.

- Exponentially many constraints, corresponding to vertices of a polyhedron.

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- Exponentially many constraints, corresponding to vertices of a polyhedron.

- Delayed Cut Generation algorithm generates vectors $\mathbf{c}$ as needed.
- For given $(\bar{\mathbf{x}}, \bar{\mathbf{y}})$, solve the separation problem (SP):

$$(\mathrm{SP}) \quad \max_{\mathbf{c} \in \mathcal{C}} \quad \mathrm{CVaR}_\alpha(\mathbf{c}^\top \hat{\mathbf{G}}(\bar{\mathbf{x}}, \bar{\mathbf{y}})) - \mathrm{CVaR}_\alpha(\mathbf{c}^\top \mathbf{Z})$$

  using an effective MIP formulation (K. and Noyan, 2016; Liu et al., 2017).

Deterministic Equivalent Formulation (DEF)

- Based on the definition of CVaR and the sufficiency to consider a finite subset of scalarization vectors in set $\mathcal{C}$, the corresponding DEF is

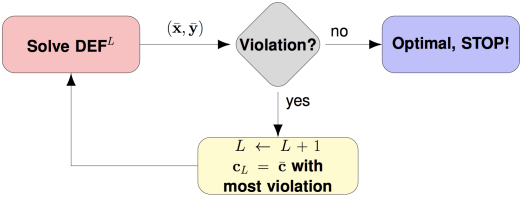
$$
\begin{aligned}
\text{(DEF)} \quad \min \quad & f(\mathbf{x}) + \sum_{s \in S} p_s \mathbf{q}_s^\top \mathbf{y}_s \\
\text{s.t.} \quad & \overbrace{\eta_l + \frac{1}{1-\alpha} \sum_{s \in S} p_s w_{sl}}^{\mathrm{CVaR}_\alpha(\hat{\mathbf{c}}_{(l)}^\top \hat{\mathbf{G}}(\mathbf{x},\mathbf{y}))} \leq \mathrm{CVaR}_\alpha(\hat{\mathbf{c}}_{(l)}^\top \mathbf{Z}), \quad \forall\, l = 1, \ldots, \bar{L}, && (1) \\
& w_{sl} \geq \hat{\mathbf{c}}_{(l)}^\top \hat{\mathbf{g}}_s(\mathbf{x}, \mathbf{y}_s) - \eta_l, \quad \forall\, s \in S,\ l = 1, \ldots, \bar{L}, && (2) \\
& w_{sl} \geq 0, \quad \forall\, s \in S,\ l = 1, \ldots, \bar{L}, \\
& \mathbf{x} \in \mathcal{X}, \quad \boldsymbol{\eta} \in \mathbb{R}^{\bar{L}}, \\
& T_s \mathbf{x} + W_s \mathbf{y}_s \geq \mathbf{h}_s, \quad \forall\, s \in S, \\
& \mathbf{y}_s \in \mathbb{R}_+^{n_2}, \quad \forall\, s \in S,
\end{aligned}
$$

  $\hat{\mathbf{g}}_s(\mathbf{x}, \mathbf{y}_s)$ is the realization of the d-dimensional random outcome vector under scenario $s \in S$.

- Exponentially many constraints, corresponding to vertices of a polyhedron.

1. Delayed Cut Generation for DEF (DCG-DEF)

- Relies on successive relaxations of the multivariate polyhedral CVaR relation, and iteratively generates cuts associated with the scalarization vectors for which the risk constraints are violated.
- Starts with a small subset of $L \ll \bar{L}$ scalarization vectors and generates more as needed.
- DEF^L: a relaxation of DEF with L scalarization vectors.

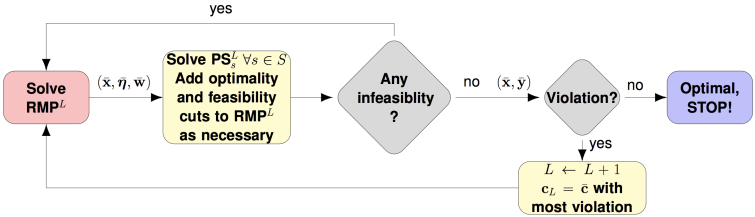


- Solving a large scale DEF becomes computationally challenging as the number of scenarios increases.

2. Delayed Cut Generation with Scenario Decomposition (DCG-SD)

- Decompose the problem over scenarios by exploiting the structure of CVaR and second-stage problems.
- RMP^L: a relaxation of DEF with $L \ll \bar{L}$ scalarization vectors and only the decision variables $(\mathbf{x}, \boldsymbol{\eta}, \mathbf{w})$
- For given solution $(\bar{\mathbf{x}}, \bar{\boldsymbol{\eta}}, \bar{\mathbf{w}})$ at an iteration with L scalarization vectors, assuming $\hat{\mathbf{g}}_s(\mathbf{x}, \mathbf{y}) = \bar{\mathbf{g}}_s\mathbf{x} + \tilde{\mathbf{g}}_s\mathbf{y}$, the subproblem for each scenario $s \in S$ becomes an LP.

$$(\mathrm{PS}_s^L) \quad \min \left\{ \mathbf{q}_s^\top \mathbf{y} \mid -\mathbf{c}_{(l)}^\top \tilde{\mathbf{g}}_s \mathbf{y} \geq \mathbf{c}_{(l)}^\top \bar{\mathbf{g}}_s \bar{\mathbf{x}} - \bar{\eta}_l - \bar{w}_{sl}, \ \forall\ l = 1, \ldots, L, \quad \mathbf{y} \in \mathcal{Y}(\mathbf{x}, \boldsymbol{\xi}_s) \right\}$$



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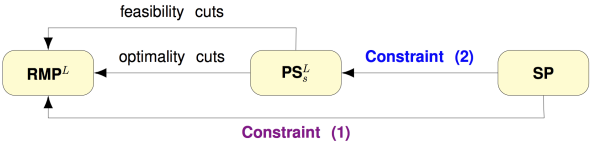
- Can be seen as a delayed column and cut generation algorithm.



Stochastic Pre-disaster Relief Network Design Problem

- Pre-positioning of relief supplies at strategic locations to improve the effectiveness of the immediate post-disaster response operations.
- Decide on the **locations and capacities of the response facilities** before a disaster strikes, when there is high level of uncertainty in supply (undamaged pre-stocked supplies), demand, and transportation network conditions (pre-disaster).
- **Distribute the relief items** to satisfy the demand across the network after uncertainty is revealed (post-disaster).

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- **Distribute the relief items** to satisfy the demand across the network after uncertainty is revealed (post-disaster).

- Multiple critical criteria: sufficiency of the delivered relief items, accessibility of the relief supplies, equity in supply allocation, etc.

Stochastic Pre-disaster Relief Network Design Problem

- Extensive literature on risk-neutral two-stage stochastic programs. (e.g., Balçık and Beamon, 2008; Döyen et al., 2012; Rawls and Turnquist, 2010; Salmerón and Apte, 2010)

- **Only a few risk-averse stochastic models for the univariate case** (e.g., Rawls and Turnquist, 2011; Noyan, 2012; Hong et al., 2015; Elci et al., 2018; Elci and Noyan, 2018).

- We extend the risk-neutral stochastic model proposed in Rawls and Turnquist (2010) by enforcing a multivariate CVaR constraint.

Problem Formulation - Criteria of Interest

Critical multiple and possibly conflicting performance criteria (Sphere Project, 2011; Vitoriano et al., 2011; Huang et al., 2012; Gutjahr and Nolz, 2016):

1. **Efficiency**: low cost
2. **Efficacy**: quick and sufficient distribution
3. **Equity**: fairness in terms of supply allocation and response times

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3. **Equity**: fairness in terms of supply allocation and response times

- We address the efficiency and efficacy using a weighted-sum based objective.
  - Minimize the expected total cost of opening facilities, demand shortages, and purchasing and shipping the relief supplies.
- We address the responsiveness (efficacy) and equity (in terms of supply allocation) via the multivariate CVaR constraints.

- 2-dimensional random vector in the CVaR benchmarking constraints.

  $\mathbf{g}^1 \rightarrow$ Maximum proportion of unsatisfied demand
  $\mathbf{g}^2 \rightarrow$ Total delivery amount-based average travel time score (ATS)

Computational Results - Case Study

- Disaster preparedness for the threat of hurricanes in the Southeastern part of the United States (Rawls and Turnquist, 2010)
- 30 demand nodes, each node is a candidate facility.
- Benchmark random outcome vector ($\mathbf{Z}$) is computed based on the current practice of Federal Emergency Management Agency (FEMA).

Computational Results - Performance of the Solution Methods

- Intel(R) Xeon(R) CPU E5-2630 processor at 2.40 GHz and 32 GB of RAM using Java and Cplex 12.6.0.
- Results averaged over 3 replications, 1 hour time limit

		DCG-DEF		DCG-SD		
α	# Sce.	Time (s) / [%gap]	L	Time (s) / [%gap]	# Cuts	L
0.99	400	2622.4	3.7	351.5	18569.7	5.3
	500	3071.3	3.0	363.3	20805.0	4.3
	600	[0.2]$^{\dagger **}$	-	662.1	27068.7	5.0
	1000	***	-	1594.2	43934.0	5.3
	1500	***	-	3450.6 [0.4]††	62436.0	4.0
0.95	400	3392.7	1.7	536.8	20633.3	4.0
	500	2753.2 [0.2]†	1.5	671.9	23045.0	3.3
	600	[0.1]$^{\dagger\dagger *}$	-	878.0	26151.3	3.3
	1000	***	-	1857.7	42229.3	3.3
	1500	***	-	3390.1 [1.4]††	52294.0	2.0
0.90	400	2263.0	1.3	623.9	19482.3	3.3
	500	1660.2 [0.3]†	1.0	1392.1	25358.0	3.3
	600	2264.3 [0.1]††	1.0	857.5 [7.9]†	30051.0	3.0

$\dagger$ ($*$) : Each dagger (asterisk) sign indicates one instance hitting the time limit with an integer (no integer) feasible solution.

Model Analysis

- **Value of risk-aversion:** Risk-averse solutions vs those of base model without the multivariate risk constraint, denoted as $\mathbf{Z}^N$.

- **Sensitivity to choice of benchmark:** Benchmark outcome vector $\mathbf{Z} = \mathbf{b} \circ \mathbf{Z}^N = (\mathrm{b}_1 Z_1^N, \mathrm{b}_2 Z_2^N)$, where $\mathbf{b}$ is a two-dimensional vector of (benchmark) control parameters.

  - Using smaller values of $\mathbf{b}$ corresponds to enforcing more demanding benchmarking constraints to ensure a better performance than $\mathbf{Z}^N$.

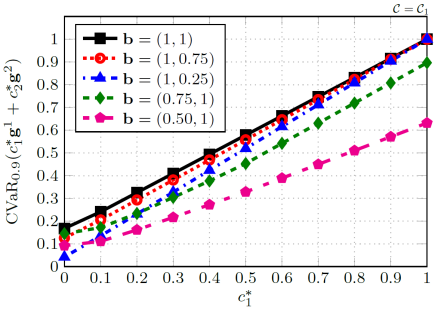
- **Sensitivity to choice of scalarization sets:** Set $\mathcal{C} = \mathcal{C}_\gamma = \{\mathbf{c} \in \mathbb{R}_+^2 : c_1 + c_2 = 1, c_2 \geq \gamma c_1\}$ for $\gamma \geq 0$.

  - The parameter γ controls the inclusiveness of different views on the relative importance of multiple criteria.

  - $\mathcal{C}_\gamma$ enlarges as γ decreases; $\mathcal{C}_{\gamma \to \infty}$ converges to a set with single element $(0, 1)$ and $\mathcal{C}_0$ corresponds to the unit simplex.

Value of incorporating risk-aversion

- Benchmark outcome vector $\mathbf{Z} = \mathbf{b} \circ \mathbf{Z}^N = (\mathrm{b}_1 Z_1^N, \mathrm{b}_2 Z_2^N)$.
- $\mathbf{Z} = \mathbf{Z}^N$ for $\mathbf{b} = (1, 1)$.
- $\mathbf{c}^* = (1, 0) \rightarrow$ univariate CVaR value w.r.t. **equity** criterion
  $\mathbf{c}^* = (0, 1) \rightarrow$ univariate CVaR value w.r.t. **responsiveness** criterion



- The risk-averse model provides better solutions than base in terms of equity and/or responsiveness measures according to the univariate CVaR-preferability.

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  - Responsiveness: $\mathbb{E}$(ATS) $\uparrow$ 9%, $\text{CVaR}_{0.9}$(ATS) $\downarrow$ 14%
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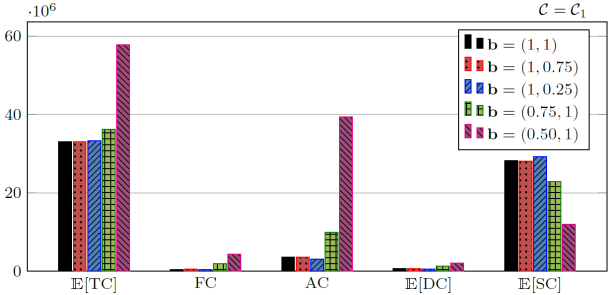
  - Efficiency: $\mathbb{E}$(total cost) $\uparrow$ 10% – No free lunch!
- Risk-averse solutions stock more inventory as benchmarks get stricter for either criterion and for larger scalarization sets.

Cost of incorporating risk-aversion under varying benchmarks **b**

Expected total cost (TC) and its components:
the total facility setup cost (FC), the total acquisition cost (AC), the total distribution cost (DC), the total demand shortage cost (SC)



- $\mathbb{E}$(TC) is not affected much by stricter responsiveness requirements ($b_2 < 1$)
- For stricter equity requirement with $b_1 = 0.75$, modest increase in $\mathbb{E}$(TC), with $\text{CVaR}_{0.9}$(prop unmet demand) $\downarrow$ 10%, $\text{CVaR}_{0.9}$(ATS) $\downarrow$ 14%

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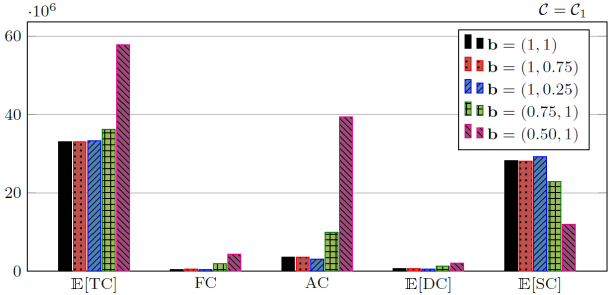

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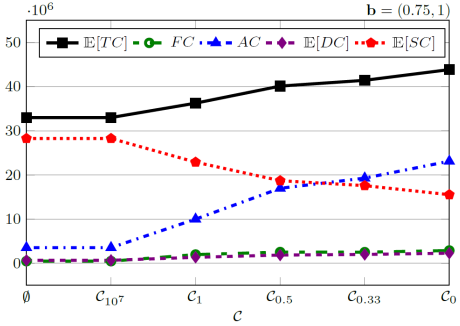
- For (even) stricter equity requirement with $b_1 = 0.50$, significant increase in $\mathbb{E}$(TC), with $\text{CVaR}_{0.9}$(prop unmet demand) $\downarrow$ 37%, $\text{CVaR}_{0.9}$(ATS) $\downarrow$ 45%

Cost of incorporating risk-aversion under varying scalarization sets $\mathcal{C}_\gamma$.

Expected total cost (TC) and its components:
the total facility setup cost (FC), the total acquisition cost (AC), the total distribution cost (DC), the total demand shortage cost (SC)



- $\mathbb{E}$(TC) increases as larger set of opinions are considered (e.g., $\mathcal{C}_0$)

Summary of Model Analysis

- Risk-averse model provides **better solutions in terms of equity and/or responsiveness** measures according to the univariate CVaR-preferability while **compromising from the expected total cost**.
    - The trade-off between equity, responsiveness and cost can be controlled via varying the benchmark, the scalarization set and the confidence level.
- The risk-averse policies tend to open more and larger facilities, and stock more inventory.
- The results demonstrate the flexibility of the proposed modeling approach to provide **a wide range of solutions** that are inclusively aligned with multiple decision makers having **different opinions on the relative importance of each criterion**.

Conclusions

- Introduce a new class of risk-averse two-stage optimization models with multivariate CVaR constraints.
  - Provide a flexible and tractable way of considering decision makers' risk preferences based on multiple stochastic criteria.
- In addition, we consider the second-order stochastic dominance (SSD)-based counterpart and provide a new computationally tractable and exact solution algorithm for this problem class.
- We propose an exact unified decomposition framework for solving these two classes of optimization problems and show its finite convergence.
- Applied proposed methods to a stochastic pre-disaster relief network design problem.
  - Our numerical results on these large-scale problems show that our proposed algorithm is computationally scalable.

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